

NAG Toolbox for MATLAB

f07gf

1 Purpose

f07gf computes a diagonal scaling matrix S intended to equilibrate a real n by n symmetric positive-definite matrix A , stored in packed format, and reduce its condition number.

2 Syntax

```
[s, scond, amax, info] = f07gf(uplo, n, ap)
```

3 Description

f07gf computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B , $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham 2002).

4 References

Higham N J 2002 *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether the upper or lower triangular part of A is stored in the array **ap**, as follows:

uplo = 'U'

The upper triangle of A is stored.

uplo = 'L'

The lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – int32 scalar

n , the order of the matrix A .

Constraint: $n \geq 0$.

3: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n symmetric, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;
 if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

Only the elements of **ap** corresponding to the diagonal elements A are referenced.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **s**(*) – double array

Note: the dimension of the array **s** must be at least $\max(1, \mathbf{n})$.

If **info** = 0 on exit, **s** contains the diagonal elements of the scaling matrix S .

2: **scond** – double scalar

If **info** = 0 on exit, **scond** contains the ratio of the smallest value of $\mathbf{s}(i)$ to the largest value of $\mathbf{s}(i)$.
 If **scond** ≥ 0.1 and **amax** is neither too large nor too small, it is not worth scaling by S .

3: **amax** – double scalar

$\max |a_{ij}|$. If **amax** is very close to overflow or underflow, the matrix A should be scaled.

4: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **s**, 5: **scond**, 6: **amax**, 7: **info**.

info > 0

If **info** = i , the i th diagonal element of A is not positive (and hence A cannot be positive-definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Further Comments

The complex analogue of this function is f07gt.

9 Example

```
uplo = 'U';  
n = int32(4);  
ap = [4.16;  
      -312000;  
      503000000000;  
      0.560000000000000001;  
      -83000;  
      0.76;  
      -0.1;  
      118000;  
      0.34;  
      1.18];  
[s, scond, amax, info] = f07gf(uplo, n, ap)
```

```
s =  
    0.4903  
    0.0000  
    1.1471  
    0.9206  
scond =  
    3.8871e-06  
amax =  
    5.0300e+10  
info =  
        0
```